



K19P 1190

Reg. No. :

Name :

III Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.)

Examination, October - 2019

(2017 Admn. Onwards)

MATHEMATICS

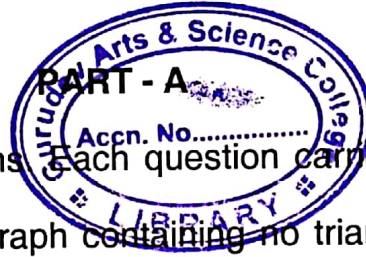
MAT 3E 01 : GRAPH THEORY

Time : 3 Hours

Max. Marks : 80

Instructions to Candidate:

- 1) Answer any 4 questions from Part - A. Each question carries 4 marks.
- 2) Answer any 4 questions without omitting any units from Part - B. Each question carries 16 marks.

I. Answer any **Four** questions. Each question carries **Four** marks.

- 1) Draw a 4-chromatic graph containing no triangles.
- 2) Prove that a set $S \subseteq V$ is an independent set of G iff $V \setminus S$ is a covering of G .
- 3) Show that the Peterson graph is 4-edge chromatic.
- 4) If G is a plane graph then prove that $\sum_{f \in F} d(f) = 2E$.
- 5) Prove that a simple graph G is n -edge connected if and only if given any pair of distinct vertices U and V of G , there are at least n edge disjoint paths from U to V .
- 6) Let U and V be two distinct vertices of a graph G . Then prove that a set F of edges of G is U - V separating if and only if every U - V path has at least one edge belonging to F .

P.T.O.



PART - B

Answer any 4 questions without omitting any Unit. Each question carries 16 marks.

UNIT - I

- II. a) Define the edge independence number and edge covering number of a graph G and prove that if minimum degree of G is greater than zero, then sum of edge covering number and edge independence number is equal to the number of vertices. (8)
- b) For any two integers $k \geq 2$ and $l \geq 2$ prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$. (8)
- III. a) If G is 4-chromatic, then prove that G contains a subdivision of K_4 . (8)
- b) If G is a connected simple graph and is neither an odd cycle nor a complete graph then prove that $\chi \leq \Delta$. (8)
- IV. a) In a bipartite graph G with $\delta > 0$ prove that the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering. (8)
- b) Define a (k, l) Ramsey graph, give one example of a Ramsey graph and show that $r(k, l) \leq \left(\frac{k+l-2}{k-1} \right)$. (8)

UNIT - II

- V. a) Prove that every planar graph is 5 vertex colourable. (8)
- b) Let G be a non planar connected graph that contain no subdivisions of K_5 or $K_{3,3}$ and has a few edges as possible, then prove that G is simple and 3 connected. (8)
- VI. a) State and prove Eulers formula for planar graph and deduce that $K_{3,3}$ is non planar. (8)
- b) Show that inner bridges avoid one another. (8)



- VII. Prove that a graph is planar if and only if it contains no subdivisions of K_5 or $K_{3,3}$ further check $K_{3,3}$ -c is planar or not. (16)

UNIT - III

- VIII. a) Show that a matching M in G is a maximum matching if and only if G contains no M augmenting path. (12)
- b) When will you say that a graph G is factorable give example of a graph G , which have 3 factors. (4)
- IX. Prove that a graph G has a perfect matching if and only if $O(G - S) \leq |S|$ for all $S \subseteq V$. (16)
- X. a) State and prove the max-flow-min-cut Theorem. (8)
- b) Let u and V be two vertices of a graph G then prove that the maximum number of edge disjoint $U-V$ paths in G equals the minimum number of edges is a $U-V$ separating set. (8)
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