



0061050

K19U 2256

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.)

Examination, November-2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B 07 MAT: Differential Equations, Laplace Transform and Fourier series

Time : 3 Hours

Max. Marks : 48

**SECTION - A**

All the 4 questions are compulsory. They carry 1 mark each. (4×1=4)

1. Solve the differential equation  $y'' = x^{-4}$ .
2. Evaluate  $(D-2)(D+1)e^{2x}$
3. Find the Laplace transform of  $e^t \cosh 3t$
4. Show that if  $f(x)$  and  $g(x)$  have period  $p$ , then  $h = af + bg$ , where  $a$  and  $b$  are constants, has period  $p$ .

**SECTION - B**

Answer any 8 questions among the questions 5 to 14. These questions carry 2 marks each. (8×2=16)

5. Show that  $2xy dx + x^2 dy = 0$  is exact and hence solve it.
6. Solve  $y' - y = e^{2x}$ .

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7. Solve the boundary value problem  $y'' + y = 0$ ,  $y(0) = 3$ ,  $y(\pi) = -3$ .
8. Define the Wronskian of two solutions  $y_1, y_2$  of second order linear homogenous equation and find the Wronskian of  $e^x$  and  $xe^x$ .
9. Solve the non homogenous equation  $y'' + 4y = 8x^2$ .
10. Find a basis of solutions for  $x^2 y'' - xy' + y = 0$ , for positive  $x$ .
11. Define the unit step function and derive its Laplace transform.
12. State the convolution theorem and find the convolution of 1 and  $t$ .
13. Find the Fourier series of  $f(x) = x + \pi$  if  $-\pi < x < \pi$  and  $f(x + 2\pi) = f(x)$ .
14. State the Fourier convergence theorem.

### SECTION - C

Answer any 4 questions among the questions 15 to 20. These questions carry 4 marks each. (4×4=16)

15. Give an example of an initial value problem, which has more than one solution.
16. State and prove the superposition principle for the homogenous linear system.
17. Solve  $y'' + 10y' + 25y = e^{-5x}$ .
18. Factor  $p(D) = D^2 + D - 6$  and solve  $p(D)[y] = 0$ .
19. Find the inverse Laplace transform of  $F(s) = \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4e^{-2s}}{s} + \frac{se^{-\pi s}}{s^2 + 1}$ .



20. Find the Fourier series of  $f(x) = |x|$ ,  $-2 < x < 2$ ,  $f(x+4) = f(x)$ .

### SECTION - D

Answer any 2 questions among the questions 21 to 24. These questions carry 6 marks each. (2×6=12)

21. Find the orthogonal trajectories of  $y = cx^2$ , where  $c$  is arbitrary.
22. Solve the differential equation  $y'' + y = \sec x$ .
23. Find the solution of  $y'' + 2y' + 2y = 5u(t - 2\pi)\sin t$ ,  $y(0) = 1$ ,  $y'(0) = 0$ .
24. Find the Fourier series of  $f(x) = \frac{x^2}{2}$ ,  $-\pi < x < \pi$ ,  $f(x + 2\pi) = f(x)$ . Hence show that  $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots = \frac{\pi^2}{12}$  and  $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$ .
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