Reg. No. : ..............................

Name : ..............................

### III Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.)
### Examination, November - 2019
### (2017 Admn. Onwards)
### CORE COURSE IN MATHEMATICS
### 3B 03 MAT : ELEMENTS OF MATHEMATICS-I

Time : 3 Hours 

Max. Marks : 48

## SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

1. State True/False, the square of an odd integer is odd.
2. Find the remainder when $x^3 - 7x - 1$ is divided by $x + 2$.
3. State Sturm's theorem.
4. The greatest common divisor of -17 and 17 is?

## SECTION-B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. If $A_m$ is a countable set for each $m \in N$ prove that the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable.

6. Prove that square of an odd integer is also an odd integer.

7. Form a polynomial equation of fourth degree with rational coefficients having one root $\sqrt{2} + \sqrt{-3}$.

8. If $\alpha, \beta, \gamma$ are the roots of $2x^3 + 3x^2 - x - 1 = 0$ find the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$.

9. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ find the value of $\sum(\alpha - \beta)^2$

10. Show that $x^5 - 2x + 7 = 0$ has at least two imaginary roots.

P.T.O.

11. Find the sum of the trigonometric series $1 - \frac{1}{2}\cos\alpha + \frac{1}{2}\frac{3}{4}\cos 3\alpha + \cdots\cdots$

12. If $\frac{a}{c}$ and $\frac{b}{c}$ with $\gcd(a,b)=1$ prove that $ab/c$

13. Prove that there is an infinite number of primes.

14. Find the remainder when $2^{50}$ is divided by 7.

## SECTION-C

Answer any **4** questions from among the questions **15 to 20**. These questions carry **4** marks each.

15. State and prove Cantor's theorem.

16. Solve $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ given that the roots are in arithmetic progression.

17. Solve the reciprocal equation $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$

18. Solve the Diophantine equation $172x + 20y = 1000$

19. Prove that the integer $53^{103} + 103^{53}$ is divided by 39.

20. Find all prime numbers that divide 50.

## SECTION-D

Answer any **2** questions from among the questions **21 to 24**. These questions carry **6** marks each.

21. a)   Prove that the Q of rational numbers is denumerable

    b)   Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

22. If $\alpha$, $\beta$, $\gamma$ are the roots of $x^3 + qx + r = 0$ find the equation whose roots are

    $\dfrac{\beta}{\gamma} + \dfrac{\gamma}{\beta}, \dfrac{\gamma}{\alpha} + \dfrac{\alpha}{\gamma}, \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ .

23. Solve $x^3 + 6x = 20$ using carden's method.

24. Using Euclidean algorithm obtain the $\gcd(826, 1890)$ and find integers x and y such that $\gcd(826,1890) = 826x + 1890y$ .

_____